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Final state interaction phases in $(B \to K\pi)$ decay amplitudes

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Abstract

A simple Regge pole model for $K\pi$ scattering explains the large phase $e^{i\delta}$ between isospin amplitudes which is observed at the D meson mass ($\delta \approx \frac{\pi}{2}$). It predicts $\delta \approx 14° - 20°$ at the B mass. Implications for $(B \to K\pi)$ decays and extensions of the model to other two-body decay channels are briefly discussed.

1. Introduction

With B-factories forthcoming, detailed checks of the precise CP-violation pattern predicted by the standard model will become possible. However it is by no means trivial to extract reliable information on CP-violation parameters from various B-decay modes. One of the problems is of course how to estimate ‘‘hadronic effects’’ such as final state interaction (FSI) phases. Although these phases are of no particular interest by themselves, they do play an important role for many potential signals of CP violation in hadronic B-decays.

The relevant question concerning these FSI phases is whether they are significantly different from 1 or not. Clearly the answer to this question depends on the hadronic channels considered. Here we will focus our attention on $(K\pi)$ channels where experimental data also exist for B decays [1]. There are two isospin invariant scattering amplitudes ($I = 1/2$ and $I = 3/2$) and the quantity one wants to estimate, as a function of energy, is $\delta(s) = \delta_3(s) - \delta_1(s)$ namely the difference between the S-wave phase shifts in the $I = 3/2$ (δ_3) and $I = 1/2$ (δ_1) amplitudes. As a matter of fact $\delta(s)$ has been measured at the D mass ($s = m_D^2$) where it is found [2] to be around $\pi/2$. Naively one does not expect such a huge FSI angle at $s = m_D^2$ to become negligible at $s = m_B^2$ but, obviously, a more quantitative argument is called for.

The main purpose of this letter is to suggest a Regge model as a general strategy for determining FSI angles [3]. Past experience with πN and $\bar{K}N$ scattering amplitudes strongly suggests that such a model should work quite well for $K\pi$ scattering over an energy range which includes the D and B meson masses.

The dominant Regge exchanges to consider in $K\pi \to K\pi$ scattering are, respectively, the Pomeron (P) and the exchange degenerate $\rho - f_2$ trajectories in the t-channel and in the u-channel the exchange degenerate $K^* - K^{**}$ trajectories. In the next section we briefly recall a few properties of these trajectories and then proceed to show in Section 3

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that with all parameters fixed phenomenologically our model automatically accounts for the observed $\delta(m_D^2) \simeq \frac{\pi}{2}$. From the known energy dependence of Regge trajectories one then readily predicts $\delta(m_B^2)$ close to 20 degrees, namely quite a sizeable FSI angle at the B mass, as naïvely expected. These are our main results.

To conclude this note we first comment on obvious implications of our results for $(B \to K\pi)$ decays and then end with several general remarks on the parametrization of any (quasi) two-body decay amplitude of the B mesons.

2. A Regge model for $(K\pi \to K\pi)$ scattering amplitudes

We take s,t,u to be the usual Mandelstam variables. In the s-channel, $(K\pi \to K\pi)$ scattering amplitudes are linear combinations of the isospin invariant amplitudes $A^s_{1/2}$ and $A^s_{3/2}$. In the t-channel $(K\overline{K} \to \pi\pi)$, we have isospin invariant amplitudes A^t_0 (isospin 0) and A^t_1 (isospin 1) and, similarly, in the u-channel $(K\pi \to \pi K)$, we define $A^u_{1/2}$ and $A^u_{3/2}$. The relations between these amplitudes are given by the crossing matrices

$$\begin{pmatrix} A^s_{1/2} \\ A^s_{3/2} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{6}} & 1 \\ \frac{1}{\sqrt{6}} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} A^t_0 \\ A^t_1 \end{pmatrix} = \begin{pmatrix} 1/3 & 4/3 \\ -2/3 & 1/3 \end{pmatrix} \begin{pmatrix} A^u_{1/2} \\ A^u_{3/2} \end{pmatrix}. \tag{1}$$

In a Regge model, s-channel amplitudes at high energy (large s) are parametrized as sums over (Regge pole) exchanges in the crossed channels: near the forward direction (t small), t-channel exchanges dominate while near the backward direction (t close to $-s$ or u small), it is the u-channel exchanges which are relevant.

The generic form, at large s, of a Regge pole exchanged in the t-channel is given by

$$-\beta(t) \frac{\tau + e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)} \left(\frac{s}{s_o}\right)^{\alpha(t)}. \tag{2}$$

In Eq. (2), $\beta(t)$ is the residue function, τ the signature $(\tau = \pm 1)$ and

$$\alpha(t) = \alpha_o + \alpha' t \tag{3}$$

is the (linear) Regge trajectory with intercept α_o and slope α'; finally s_o is a scale factor usually taken as 1 GeV^2. For a Regge pole exchanged in the u-channel, the generic form is similar to Eq. (2) but with the variable t replaced by u.

The leading trajectory (highest intercept) is the so-called Pomeron (P). It has the quantum numbers of the vacuum $(I=0, \tau = +1)$ and its exchange describes ''diffractive scattering''. The Pomeron always contributes to elastic scattering and describes quite well the bulk of hadronic differential cross-sections over a wide energy range.

In the energy interval which is of interest to us here, namely

$$3\ \mathrm{GeV}^2 \lesssim s \lesssim 35\ \mathrm{GeV}^2 \tag{4}$$

a very simple but excellent phenomenological parametrization of the Pomeron trajectory and residue function is given by

$$\alpha_P(t) = 1 \tag{5}$$

and

$$\beta_P(t) = \beta_P(0) e^{b_P t} \tag{6}$$

with

$$2.5\ \mathrm{GeV}^{-2} \lesssim b_P \lesssim 3\ \mathrm{GeV}^{-2} \tag{7}$$

obtained from fits [4] to elastic πp, pp and Kp differential cross-sections (using factorization). As a result the Pomeron contribution to A^t_0 now reads $(s_o = 1\ \mathrm{GeV}^2)$

$$A_P = i\beta_P(0) e^{b_P t} s. \tag{8}$$

The next trajectories to consider are the $\rho - f_2$ trajectories in the t-channel and the $K^* - K^{**}$ trajectories in the u-channel. The ρ trajectory has $T=1, \tau = -1$ while the f_2 trajectory has $T=0, \tau = +1$; similarly the K^* trajectory has $T=1/2, \tau = -1$ while the K^{**} trajectory has the opposite signature. Because of the absence of exotic resonances (no $K\pi$ resonances with $I=3/2$), the ρ and f_2 trajectories as well as the $K^* - K^{**}$ ones must be exchange degenerate. Specifically this means that the ρ and f_2 trajectories coïncide

$$\alpha_\rho(t) = \alpha_{f_2}(t) \cong \tfrac{1}{2} + t \tag{9}$$

and that their residues are related i.e.

$$\frac{\beta_{f_2}(t)}{\sqrt{6}} = \frac{\beta_\rho(t)}{2}. \tag{10}$$

Similarly, for the $K^* - K^{**}$ trajectories (in the $SU(3)$-limit)

$$\alpha_{K^*}(u) = \alpha_{K^{**}}(u) \cong \tfrac{1}{2} + u \tag{11}$$

and

$$-\beta_{K^*}(u) = \beta_{K^{**}}(u). \tag{12}$$

Eqs. (9), (10) and Eqs. (11), (12) guarantee that the non diffractive imaginary part of $A^s_{3/2}$ vanishes. They used to be called ‘‘duality constraints’’ [5].

We neglect lower lying trajectories such as the $\rho'(I=1,\tau=-1)$ and the $f_0(I=0,\tau=+1)$ in the t-channel as well as their $SU(3)$ partners in the u-channel. Were we to include them they should also be taken as exchange degenerate.

It is customary to write the residue function of the ρ trajectory as

$$\beta_\rho(t) = \frac{\overline{\beta}_\rho(t)}{\Gamma(\alpha(t))}. \tag{13}$$

Since $\Gamma(\alpha(t))\sin\pi\alpha(t)$ is a very smooth function of t, no harm is done in using at small t the approximations

$$\Gamma(\alpha(t))\sin\pi\alpha(t) \approx \Gamma(\alpha(0))\sin\pi\alpha(0) = \sqrt{\pi}, \tag{14}$$

$$\overline{\beta}_\rho(t) \approx \overline{\beta}_\rho(0), \tag{15}$$

and in writing the ρ trajectory contribution to A^t_1 as

$$A_\rho(s,\text{small } t) = \frac{\overline{\beta}_\rho(0)}{\sqrt{\pi}}\left(1 + i\exp(-i\pi t)\right)s^{0.5+t}. \tag{16}$$

An exactly similar reasoning gives for the f_2 trajectory contribution to A^t_0

$$A_{f_2}(s,\text{small } t) = \sqrt{\tfrac{3}{2}}\,\frac{\overline{\beta}_\rho(0)}{\sqrt{\pi}}\left(-1 + i\exp(-i\pi t)\right)s^{0.5+t}, \tag{17}$$

while for the K^* and K^{**} trajectories contributions to $A^u_{1/2}$ one writes

$$A_{K^*}(s,\text{small } u) = \frac{\overline{\beta}_{K^*}(0)}{\sqrt{\pi}}\left(1 + i\exp(-i\pi u)\right)s^{0.5+u}, \tag{18}$$

$$A_{K^{**}}(s,\text{small } u) = -\frac{\overline{\beta}_{K^*}(0)}{\sqrt{\pi}}\left(-1 + i\exp(-i\pi u)\right)s^{0.5+u}, \tag{19}$$

with

$$\overline{\beta}_{K^*}(0) = \tfrac{3}{4}\overline{\beta}_\rho(0) \tag{20}$$

in the $SU(3)$ limit.

Putting everything together and using the crossing matrices given in Eq. (1), our Regge model for $K\pi$ scattering is now completely defined by the amplitudes

$$A^s_{1/2}(s,\text{small } t) = \frac{i}{\sqrt{6}}\beta_P(0)e^{b_P t}s + \frac{\overline{\beta}_\rho(0)}{2\sqrt{\pi}}s^{0.5+t} + \frac{3i\overline{\beta}_\rho(0)}{2\sqrt{\pi}}e^{-i\pi t}s^{0.5+t}, \tag{21a}$$

$$A^s_{1/2}(s,\text{small } u) = \frac{\overline{\beta}_\rho(0)}{2\sqrt{\pi}}s^{0.5+u}, \tag{21b}$$

and

$$A^s_{3/2}(s,\text{small } t) = \frac{i}{\sqrt{6}}\beta_P(0)e^{b_P t}s - \frac{\overline{\beta}_\rho(0)}{\sqrt{\pi}}s^{0.5+t}, \tag{22a}$$

$$A^s_{3/2}(s,\text{small } u) = -\frac{\overline{\beta}_\rho(0)}{\sqrt{\pi}}s^{0.5+u}. \tag{22b}$$

3. S-wave rescattering phases

The remaining task is now to extract from Eqs. (21), (22) the $\ell=0$ partial wave amplitudes $a_{1/2}(s)$ and $a_{3/2}(s)$. Neglecting π and K masses, we have, up to irrelevant real factors

$$a_I(s) \propto \int_{-s}^{0} dt A^s_I(s,t). \tag{23}$$

From the physical ideas underlying Eqs. (21), (22) it is clear that outside the forward and backward regions, the integral in Eq. (23) gives a negligibly small contribution to $a_I(s)$. We thus write

$$a_I(s) \propto \int_{t_o}^{0} dt A_I^s(s, \text{small } t) + \int_{u_o}^{0} du A_I^s(s, \text{small } u). \tag{24}$$

With the explicit expressions given in Eqs. (21), (22), the integrals in Eq. (24) are trivial to perform. Furthermore, the integrated contributions at the t_o and u_o boundaries (around 1 GeV^2) are considerably smaller than at the boundary 0 of both integrals in Eq. (24). Neglecting these contributions, one thus obtains

$$a_{1/2}(s) = \frac{i}{\sqrt{6}} \frac{\beta_P(0)}{b_P} s + \frac{\overline{\beta}_\rho(0)}{\sqrt{\pi}} \frac{1}{\ln s} s^{1/2} + \frac{3i}{2\sqrt{\pi}} \overline{\beta}_\rho(0) \frac{\ln s + i\pi}{(\ln s)^2 + \pi^2} s^{1/2} \tag{25}$$

and

$$a_{3/2}(s) = \frac{i}{\sqrt{6}} \frac{\beta_P(0)}{b_P} s - 2 \frac{\overline{\beta}_\rho(0)}{\sqrt{\pi}} \frac{1}{\ln s} s^{1/2} \tag{26}$$

from which the $\tan(\delta_I) = \frac{\mathrm{Im}\, a_I(s)}{\mathrm{Re}\, a_I(s)}$ are straightforward to compute. Note that both $\tan(\delta_1)$ and $\tan(\delta_3)$ depend on one single phenomenologically determined parameter namely

$$x = \frac{\sqrt{\pi}\, \beta_P(0)}{b_P \overline{\beta}_\rho(0)}. \tag{27}$$

From fits [6] to πp, pp and Kp total cross sections in the energy range given in Eq. (4) (again using factorization), we find

$$\frac{\sqrt{\pi}\, \beta_P(0)}{\overline{\beta}_\rho(0)} = 2.9 \pm 0.2. \tag{28}$$

From Eq. (7), we thus conclude that x is close to one

$$x = 1.07 \pm 0.17. \tag{29}$$

Similar results are obtained using the fits given in Ref. [7] for a larger energy range.

With these values for x, the range for the FSI angle at the D mass is *calculated* to be

$$\delta(m_D^2) \equiv \delta_3(m_D^2) - \delta_1(m_D^2) = (85 \pm 6)^\circ \tag{30}$$

in spectacular agreement with the recent analysis of CLEO data [2]

$$\delta(m_D^2) = (96 \pm 13)^\circ. \tag{31}$$

We stress that both the analysis of CLEO data and our calculation are based on the quasi-elastic approximation.

At the B mass, we predict a sizeable angle close to 20 degrees, namely

$$\delta(m_B^2) = (17 \pm 3)^\circ. \tag{32}$$

Before commenting on our prediction for $\delta(m_B^2)$, it may be worthwhile to point out a few facts about our calculation of $\delta(s)$

- it is a no-parameter calculation: x is determined from the data on total cross-sections [6] and $\frac{d\sigma}{dt}$'s [4];
- in performing our calculation of $\delta(s)$, we have made several approximations a.o. we neglected lower trajectories as well as the intermediate region in the S-wave projection integral. These approximations are certainly sound from a phenomenological point of view and they become better and better as s increases. At the D mass we do not believe that our end result should be trusted to better than 10–20% but in any case, agreement with the data remains excellent;
- the calculations presented here for $K\pi$ scattering can of course be repeated for $\pi\pi$ or $K\overline{K}$ scattering. A detailed account and discussion of these calculations will be presented elsewhere [8]. Here we simply point out that the results of both calculations are once again in excellent agreement with the data available [2] at the D mass : $\delta^{\pi\pi}$ is found to be around $\pi/3$ and $\delta^{K\overline{K}}$ around $\pi/6$. These results considerably strengthen our confidence in a simple Regge parametrization of hadronic scattering amplitudes.

4. Conclusions

The main results of this letter are given by Eqs. (30)–(32) and can be summarized as follows: a Regge model for $K\pi$ scattering explains the large S-wave rescattering phase difference δ observed at the D meson mass namely $\delta(m_D^2) \approx \frac{\pi}{2}$, and predicts $\delta(m_B^2) \approx 20°$.

Such a sizeable FSI angle at the B meson mass leads to important implications for $B \to K\pi$ decays [9]: it invalidates the Fleischer-Mannel bound [10] on the Cabibbo-Kobayashi-Maskawa angle γ and implies a potentially large CP asymmetry, a, in ($B^{\pm} \to K\pi^{\pm}$) decays:

$$a \approx 4(\sin\gamma)\%. \tag{33}$$

Strong interaction hadronic phases can be parametrized a la Regge for *any* (quasi) two body decay mode of the B meson ($\pi\pi, K\bar{K}$ as already mentioned, but also $\pi\rho, K^{*}\pi, K^{*}\rho$, etc.).

The fact that our quasi-elastic treatment of the scattering amplitudes for $K\pi, \pi\pi$ and $K\bar{K}$ agrees so well with the data at the D meson mass is a strong argument for neglecting inelastic effects on hadronic phases.

In view of the previous comments, a general parametrization for all two-body decay modes of the B mesons naturally suggests itself. The decay amplitude can be written as a sum of reduced matrix elements $\langle\langle B | H_W | (M_1 M_2), I \rangle\rangle$ of the effective weak hamiltonian, multiplied by the appropriate hadronic FSI phases e^{δ_I}. These reduced matrix elements are in general complex numbers which can be systematically calculated in terms of tree-level, colour suppressed, penguin, exchange or annihilation quark diagrams. Of course, no isospin violating ‘‘scattering phases’’ are allowed between these diagrams and furthermore, as already shown elsewhere [9], classes of diagrams which would naïvely be excluded can reappear due to factors of the type $(1 - e^{i\delta_I})$. On the other hand, penguin diagrams can provide an absorptive (i.e. imaginary) component to the reduced matrix elements [11]. But these imaginary parts are very model-dependent and probably quite small. Therefore we suggest [12], as a first approximation to simply ignore these ‘‘quark phases’’ whenever the hadronic phases are sizeable. This was assumed in Ref. [9]. This happens to be the case for $B \to K\pi$ decays.

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